LETTER

Low-Complexity ICI Cancellation in Frequency Domain for OFDM Systems in Time-Varying Multipath Channels*

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SUMMARY In this letter, we propose a partial minimum mean-squared error (MMSE) with successive interference cancellation (PMMSESIC) method in frequency domain to mitigate ICI caused by channel variation. Each detection, the proposed method detects the symbol with the largest received signal-to-interference-plus-noise ratio (SINR) among all the undetected symbols, using an MMSE detector that considers only the interference of several neighborhood subcarriers. Analysis and simulations show that it outperforms the MMSE method at relatively high Eb/N0 and its performance is close to the MMSE with successive detection (MMSESD) method in relatively low Doppler frequency region.

key words: time-varying multipath channels, orthogonal frequency division multiplexing (OFDM), intercarrier interference (ICI), ICI cancellation

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a promising technique for high-speed data transmission in wireless mobile communications [1], [2]. However, in mobile radio environment, multipath channels are usually time-varying. Time variation within an OFDM block period leads to a loss of subcarrier orthogonality, resulting in intercarrier interference (ICI) and performance deterioration.

To combat the ICI caused by channel variation, many approaches have been proposed, e.g., polynomial cancellation coding (PCC) [3], self-cancellation scheme [4], time domain filtering [5], minimum mean-squared error (MMSE) [6], MMSE with successive detection (MMSESD) [6]. The approaches in [5] and [6] have good performance but require \( O(N^3) \) computational complexity, where \( N \) denotes the subcarrier number of an OFDM block. The techniques proposed in [3] and [4] can significantly reduce the ICI but need to modulate the same data symbol onto a group of subcarriers with the predefined weighting coefficients, which means a substantial waste of spectral bandwidth.

In this letter, we propose a frequency-domain partial minimum mean-squared error (MMSE) with successive interference cancellation (PMMSESIC) method with \( O(N^2) \) complexity to mitigate the ICI caused by channel variation. The method exploits successive interference cancellation (SIC) technique to obtain significant gain and also uses a partial MMSE detector to avoid large matrix inversion. It will be shown that it can effectively reduce the ICI with low complexity. Throughout the letter, full channel knowledge is assumed.

2. Signal Model

In a discrete-time baseband equivalent OFDM system, \( N \) input symbols are transformed into an \( N \)-point symbol block \( X = [X_0, X_1, \ldots, X_{N-1}]^T \) by the serial-to-parallel converter. Then it is transformed into an \( N \)-point time-domain sample block \( x = [x_0, x_1, \ldots, x_{N-1}]^T \) by the normalized Inverse Fast Fourier transform (IFFT), and the \( n \)th sample can be expressed as

\[
x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}, 0 \leq n \leq N - 1.
\]

After adding the cyclic prefix (CP), which is a copy of the last samples of the IFFT output, the samples are serially transmitted over a noisy multipath time-varying channel. The received signal can be written as

\[
y_k = \sum_{l=0}^{n-1} h_{k,l} x_{k-l} + w_k, 0 \leq k \leq N - 1,
\]

where \( h_{k,l} \) represents the impulse response of time-varying multipath channels at lag \( l \) and instant \( k \), \( w \) denotes the channel maximum delay spread and \( w \) is the Additive White Gaussian noise with variance \( \sigma^2 \) and independent of the input symbols. The demodulated signal in frequency domain is obtained by taking the \( N \)-point normalized FFT of \( y_k \) as

\[
Y_m = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} y_k e^{-j2\pi mk/N}, 0 \leq m \leq N - 1.
\]

Substituting (1) and (2) into (3), we have
\[ Y_m = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \sum_{l=0}^{v-1} h_{k,l} \left( \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi(m-k)n/N} \right) + w_m e^{-j2\pi m/N} \]

\[ = \sum_{n=0}^{N-1} H_{m,n} X_n + W_m, \quad 0 \leq m \leq N - 1, \quad (4) \]

where \( H_{m,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \sum_{l=0}^{v-1} h_{k,l} e^{-j2\pi(l-k)n/N} e^{-j2\pi mk/N} \) and \( W_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} w_n e^{-j2\pi m/N} \).

### 3. Symbol Energy Distribution and ICI Analysis

In order to analyze the symbol energy distribution and the ICI on one subcarrier, we rewrite (4) in the following form,

\[ Y_m = H_{m,n} X_m + \sum_{n=0, n \neq m}^{N-1} H_{m,n} X_n + W_m, \quad 0 \leq m \leq N - 1. \quad (5) \]

Note that \( H_{m,m} X_m \) is the desired ICI-free term and \( \sum_{n=0, n \neq m}^{N-1} H_{m,n} X_n \) represents the ICI term. So the energy of leaked to the \( m \)-th subcarrier can be found as

\[ P_{m,n} = E[|H_{m,n} X_n|^2] = E_s E[H_{m,n} H_{m,n}^*] \]

\[ = E_s \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{v-1} \sum_{k'=0}^{N-1} \sum_{l'=0}^{v-1} \left( E(h_{k,l} h_{k',l'}) \right) e^{-j2\pi(l-l')/N} e^{-j2\pi(k-k')/N}, \quad (6) \]

where \( E_s \) is the symbol energy and the superscript * stands for conjugate. In wide-sense stationary uncorrelated scattering (WSSUS) Rayleigh fading channel based on the Jake’s model [7], \( E[h_{k,l} h_{k',l'}^*] = J_0(2\pi f_d|k-k'|T_c)\sigma_1^2 \delta(l-l') \), where \( J_0(\cdot) \) is the zeroth-order Bessel function of the first kind, \( f_d \) is the maximum Doppler frequency, \( T_c \) is the OFDM sample duration, \( \sigma_1^2 \) is the variance of \( h_{k,l} \) with \( \sum_{l=0}^{v-1} \sigma_1^2 = 1 \). Then (6) can be simplified as

\[ P_{m,n} = \frac{E_s}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{v-1} J_0(2\pi f_d|k-k'|T_c) e^{-j2\pi(k-k')/N} \]

\[ = P_q, \quad (7) \]

where \( q = n - m \). Using (7), the energy of \( X_n \) distributed to subcarriers \( n - L \) to \( n + L \) can be expressed as

\[ \phi_L = \sum_{q=-L}^{L} P_q = \frac{E_s}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{v-1} J_0(2\pi f_d|k-k'|T_c) \sum_{q=-L}^{L} e^{-j2\pi(k-k')q/N} \]

\[ \left. \right| \sum_{q=-L}^{L} e^{-j2\pi(k-k')q/N} \right|, \quad (8) \]

The normalized symbol energy distribution \( \phi_L / E_s \) is depicted in Fig. 1. From it we can obtain the fact that most of a symbol energy spreads over itself and its several neighborhood subcarriers when \( f_d T_c < 1 \), e.g., when \( f_d T_c = 0.1 \), more than 99% of the \( X_n \)’s energy is distributed on itself and its two neighborhood subcarriers. If the Doppler frequency increases, more symbol energy leaks to its neighborhood subcarriers, e.g., when \( f_d T_c = 0.9 \), more than 97% of the \( X_n \)’s energy is distributed on the itself and its eight neighborhood subcarriers.

When \( m \neq n \), \( H_{m,n} X_n \) denotes the interference of \( n \)-th subcarrier to the \( m \)-th subcarrier. Then (7) is the ICI power of \( X_n \) to the \( m \)-th subcarrier. From the analysis above, we can conclude that the ICI on one subcarrier mostly comes from only several neighborhood subcarriers. In [8], they gave the lower bounds on partial energy distribution and on partial ICI for the continuous signal model, which also proved this conclusion.

### 4. ICI Cancellation Scheme

In [6], the MMSESD method to mitigate ICI is based on all FFT output symbols. When the number of subcarriers is large, this method requires large matrix inversion, so the computational complexity is very high. In this section, we will exploit the fact that the ICI on one subcarrier mainly comes from only several neighborhood subcarriers to develop a low-complexity PMMSESIC method in frequency domain to mitigate ICI.

#### 4.1 ICI Cancellation Scheme

Compared with the MMSED method, we first detect the symbol \( X_k \) with the highest pre-detection signal-to-interference-plus-noise ratio (SINR) among the undetected symbols other than post-detection SINR. That is we first detect \( X_k \) which satisfies the following condition

\[ \arg \max_k \text{SINR}_k = \arg \max_k \frac{|H_{k,k}|^2 E_s}{E_s \sum_{n=0}^{N-1} |H_{k,n}|^2 + \sigma_1^2}. \quad (9) \]

The detailed detection procedure is described as follows.

Step 1: According to (9), the SINR of all the undetected
symbols in one OFDM block are calculated. From them we choose the symbol with the largest SINR, assuming $X_k$.

Step 2: Detecting the symbol $X_k$ basing on partial MMSE. Let $K = 2L + 1$, define $\rho_k$ as a $K$-dimensional column vector, and let $\rho_{ki}$ denote its $i$th element which satisfies $\rho_{ki} = (k - L - 1 + i) \mod N, 0 \leq k \leq N - 1, 0 \leq i \leq K - 1$ under the condition $\rho_{ki} \geq 0$. Define $X_k = X(\rho_k)$, $Y_k = Y(\rho_k)$, $W_k = W(\rho_k)$ and $H_k = H(\rho_k, \rho_k)$, where $X(\rho_k)$, $Y(\rho_k)$ and $W(\rho_k)$ denote the subvectors within the column vectors $X$, $Y$ and $W$, defined by the index vector $\rho_k$. and the index vector of desired rows in $\rho_k$ and the index vector of desired columns in $\rho_k$. Then we have

$$Y_k = H_k X_k + W_k, 0 \leq k \leq N - 1$$

(10)

Based on (10), the transmitted block $X_k$ is detected with the partial MMSE detector $G_k = (H_k^H H_k + \sigma^2 I_K)^{-1} H_k^H$. We obtain

$$\hat{X}_k = G_k Y_k = G_k H_k X_k + G_k W_k$$

(11)

where $\hat{X}_k = [\hat{X}_{k0}, \hat{X}_{k1}, \ldots, \hat{X}_{kL}]^T$. Then we get the estimation of the transmitted symbol $X_k$ through $\hat{X}_k = \hat{X}_{k0}$.

Step 3: $\hat{X}_k$ = hard decision of $\hat{X}_k$.

Step 4: Updating the received signal vector $Y$ and the channel matrix $H$ by $Y = Y - H_{\cdot k} \hat{X}_k$ and $H_{\cdot k} = 0$ successively, where $H_{\cdot k}$ represents the $k$th column vector within $H$.

Step 5: Repeating step 1 to step 4 until all the symbols of the current OFDM block are detected completely.

4.2 Computational Complexity Analysis

Considering the $l$th detection, to obtain the symbol with largest pre-detection SINR among the undetected symbols requires $N - l$ compare operations and $N - l$ multiplicative operations; to detect it, we need $O(K^3)$ operations to calculate the partial MMSE detector $G_k$. Therefore, to detect the whole OFDM block demands $N(N - 1)/2$ compare operations, $N(N - 1)/2$ multiplicative operations and $O(K^3 N)$ operations to calculate every corresponding partial MMSE detector. When $N$ becomes very large, e.g., $N = 1024$ or $N = 2048$, then $K \ll N$, the partial MMSE detector’s computational complexity is linear in the OFDM block size $N$, and the major computation is to search the symbol with largest pre-detection SINR, so the total complexity of our method is $O(N^2)$. It is obvious that the complexity of our approach is lower than MMSE and MMSESD [6], which are $O(N^3)$ and $O(N^4)$ respectively.

5. Simulations

To demonstrate the effectiveness of our proposed method for time-varying multipath channels, the following simulations were performed. We consider an OFDM system with the number of subcarriers $N = 64$, and the length of the cyclic prefix is 6 samples. The uncoded QPSK constellation is adopted. We use a tap-delay-line WSSUS channel model of two paths with an exponential delay power spectrum. Each channel tap is modeled as a complex Gaussian random process independently generated with the Jakes’ Doppler spectrum [7]. The relative delay of the first tap is zero. The power of the second path is 5 dB smaller than the first one and the relative delay is 5 samples.

Figure 2 and Fig. 3 show the BER performance comparison when MMSE, MMSESD and PMMSEIC are used to reduce the performance degradation due to time variation of multipath fading channels, with $f_d T_s = 0.05$, $L = 3$, 5 and $f_d T_s = 0.1$, $L = 3$, 4, 5 respectively. From them, we can see that at low Eb/N0, the performance of the three methods is very close; while at relatively high Eb/N0, our approach significantly outperforms the MMSE method, e.g., at BER = $3.0 \times 10^{-8}$, when $f_d T_s = 0.05$, $L = 3$ and $f_d T_s = 0.1$, $L = 3$, the gain of the performance of our approach PMMSEIC to MMSE is 4 dB and 6 dB respectively. In relatively low Doppler frequency region, the performance of our approach is very close to MMSESD, e.g., at BER = $10^{-4}$, when $f_d T_s = 0.05$, $L = 3$, and 5 the loss of the performance of our approach PMMSEIC to MMSE is not more than 0.4 dB and 0.3 dB respectively. When Doppler frequency increases, the loss becomes big, e.g., at BER = $10^{-4}$, when $f_d T_s = 0.1$,
When $L = 3, 4$ and $5$, the loss is $3.4$ dB, $2.6$ dB and $2.3$ dB respectively, but from the computational complexity analysis, the complexity of our approach is far lower than MMSESD.

Figure 2 and Fig. 3 also illustrate the effect of different Doppler frequency on the performance of the MMSE, MMSESD and PMMSEIC detectors. For our proposed method, it can be seen from the figures that, at moderate Eb/N0, when the Doppler frequency increases, the performance improves. E.g., at $\text{BER}=3.0 \times 10^{-4}$, when $L = 3$, comparing the performance when $f_d T_s = 0.1$ and $f_d T_s = 0.05$, the gain is about $1.4$ dB. This is because as the Doppler frequency increases, more time diversity becomes available. This also happens for the MMSESD method. The analysis of time diversity due to time-variation of the channel is given in [6] when they proposed the MMSESD detection method in Sect. 3. But at high Eb/N0, for our method, as the Doppler frequency increases, the performance decreases. The reason is that, at high Eb/N0, the residual interference determines the BER performance. The larger Doppler frequency is, the more residual interference is left. The gain from time diversity is overwhelmed by the residual interference. Moreover, in Fig. 2 and Fig. 3, the BER performance of our proposed method is plotted as a function of $L$. It can be seen from the figures that when $L$ is big enough, e.g., $f_d T_s = 0.05$, $L = 3$ and $f_d T_s = 0.1$, $L = 4$, with the increase of $L$, the performance improvement is not obvious, and bigger $L$ means higher complexity. So in order to obtain a good tradeoff between performance and complexity, $L$ should be selected appropriately.

6. Conclusions

Based on the fact that the ICI on one subcarrier mainly comes from several neighborhood subcarriers when $f_d T_s < 1$, we proposed an equalization technique in frequency domain to mitigate the ICI caused by time-varying multipath channels. It introduces the SIC technique to obtain significant gain and uses the partial MMSE detector to avoid large matrix inverse, so it is capable of mitigating ICI effectively with low complexity. It outperforms the MMSE method at relatively high Eb/N0, and its performance is close to the MMSESD method when the Doppler frequency is relatively low. At moderate Eb/N0, its performance improves when the Doppler frequency increases. Its complexity is the lowest among the three methods.

References