Kalman-filter-based channel estimation for orthogonal frequency-division multiplexing systems in time-varying channels

M. Huang, X. Chen, L. Xiao, S. Zhou and J. Wang

Abstract: A low-complexity Kalman-filter-based channel estimation method for orthogonal frequency-division multiplexing systems is proposed. This method belongs to the pilot-symbol-aided parametric channel estimation method in which the channel responses are characterised as a collection of sparse propagation paths. Because of the slow variation of the signal subspace in the channel samples’ correlation matrix, the estimation of channel parameters is translated into an unconstrained minimisation problem. Then, in order to solve this optimisation problem, a subspace tracking by Kalman filter is carried out, which is characterised in that the state equation and the measurement equation are constructed upon the constant signal subspace. Further, this Kalman-filter-based method is extended to the multi-antenna scenarios efficiently. Simulation results show that the proposal can effectively track the time variations in both the block fading channels and the Doppler frequency spread channels.

1 Introduction

Recently, orthogonal frequency-division multiplexing (OFDM), because of its high frequency-spectrum efficiency and strong ability to mitigate multi-path propagation, is regarded as one of the key technologies for the next generation mobile communication systems – called by 4G systems [1]. Because of the request of high mobility for 4G systems, the signals at OFDM receivers would experience fast time-varying channel environment and suffer from strong Doppler frequency spread. This implies the need for receivers to exactly estimate and real-time track the variation of multi-path channels with low complexity [2].

In OFDM systems, the parametric channel estimation methods have been studied extensively. In these methods, the radio channels are modelled by a few dominant sparse paths [3]. Accordingly, the parameters of channel paths, including delays and complex amplitudes, are estimated before the stage of interpolating over the entire frequency-time grid. In current studies, the channel parameters are obtained via subspace analysis methods [4, 5] or Wiener filter [6]. However, since these methods need to cumulate a large amount of samples, they cannot track the variation of fast fading channels. In [7, 8], a subspace tracking method is developed. This method mainly employs a QR decomposition based recursive least-squares (RLS) adaptive filter to track the delay subspace, that is, the signal subspace of channel samples’ correlation matrix. Conventional approaches for the QR decomposition, such as Givens rotations and Householder transforms, all have the complexity of $O(N^3)$, for an $N \times N$ matrix. Hence, this QR-RLS method consumes the complexity as high as $O(Npr^2)$, where $N_p$ is the number of pilot subcarriers and $r$ is the number of channel paths. Contrarily, because of its low complexity compared with matrix inversions and decompositions, Kalman-filter is used to track the channel variation in [9–11]. However, these schemes do not take into account the low-dimensioned signal subspace, that is, they operate upon high-dimensioned channel matrix, which cannot improve the estimation performance with the help of the knowledge about the channel characteristics.

We apply the signal subspace tracking by Kalman-filter within the pilot-symbol aided parametric channel estimation structure. Here, the Kalman-filter is balanced on the slow-variation character of delays, not of amplitudes in the multi-path channels. On the one hand, through the adaptive tracking of a low-dimensioned signal subspace, the accuracy of estimation is greatly improved; on the other hand, the application of Kalman-filter provides fast convergence with as low complexity as $O(Npr)$. Compared with the existing methods in the literatures, low dimension and low complexity are compatible in this method. In addition, the multi-antenna scenarios are also taken into account. In this case, the channel estimation of each pair of transmit and receive antennas should be carried out. Here in our approach, the phases of pilot symbols in different transmit antennas are circled with different angles in the frequency domain, so that the subspaces of these transmit antennas can be orthogonal and hence tracked simultaneously by the Kalman-filter based method.

This paper is organised as follows. In Section 2, the system model is established. In Section 3, the proposed Kalman-filter based channel estimation method is presented. The extension to the multi-antenna scenarios is considered in Section 4. The simulation results are given in Section 5 to evaluate their performances in time-varying channels. Finally, some conclusions are drawn in Section 6.

Throughout the paper, the symbols $(\cdot)^T$ and $(\cdot)^H$ denote the matrix transpose and conjugate transpose, respectively. $\text{norm}(\cdot)$ denotes the Euclidean norm of a vector. $E(\cdot)$

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denotes the expectation operator. \( I_N \) stands for an \( N \times N \) identity matrix. \( \mathbb{N} \) and \( C \) represent the sets of natural numbers and complex numbers, respectively.

## 2 System model

We consider an OFDM system that employs \( N \) subcarriers. The system bandwidth is \( B = 1/T \), where \( T \) is the duration of one time-chip. And the duration of one OFDM symbol is \( T_s = NT + T_{CP} \), where \( T_{CP} \) is the duration of cyclic prefix (CP) for every OFDM symbol.

The time-varying multi-path channel is modelled as a collection of paths, that is,

\[
h(t, \tau) = \sum_{l=0}^{L-1} a_l(t) \delta(\tau - \tau_l)
\]

Therein the \( l \)th path can be parameterised with a complex amplitude \( a_l(t) \) and a delay \( \tau_l(t) \). Here, two assumptions about them are given.

**Assumption 1:** \( \max_l(\tau_l) < T_{CP} \), so that no inter-symbol interference (ISI) exists.

**Assumption 2:** \( a_l(t) \) and \( \tau_l(t) \) are quasi-static, that is, constant during the transmission of one OFDM symbol duration \( T_s \), and variable on a symbol-by-symbol basis \([7]\).

Define \( a_{n,l} \triangleq a_l(T_n) \), \( \tau_{n,l} \triangleq \tau_l(t) \). Because of Doppler frequency spread, \( a_{n,l} \) varies as fast as \([12]\)

\[
E[a_{n,l}^* a_{n'_l}] = J_0(2\pi f_d |n_1 - n_2| T_s) \sigma_l^2
\]

where \( J_0(.) \) is the zeroth-order Bessel function of the first kind, \( f_d \) is the maximum Doppler frequency spread, \( \sigma_l^2 \) is the variance of \( a_{n,l} \) with \( \sum_{l=0}^{L-1} \sigma_l^2 = 1 \). Obviously, with \( f_d \) increasing, the correlation between \( a_{n,l} \) and \( a_{n'_l} \) decreases.

The channel response at OFDM symbol \( n \) and subcarrier \( k \) is represented as

\[
H_n(k) = \sum_{l=0}^{L-1} a_{n,l} e^{-j2\pi k \tau_{n,l}/NT} \quad k = 0, 1, \ldots, N - 1
\]

Let

\[
\alpha_n = [a_{n,0} \quad a_{n,1} \quad \ldots \quad a_{n,L-1}]^T
\]

\[
\tau_n = [\tau_{n,0} \quad \tau_{n,1} \quad \ldots \quad \tau_{n,L-1}]^T
\]

\[
H_n = [H_n(0) \quad H_n(1) \quad \ldots \quad H_n(N - 1)]^T
\]

and \( N \times L \) Vandermonde matrix \( W(\tau_n) = [e^{-j2\pi k \tau_{n,l}/NT}]_{l=0}^{L-1} \), then, rewrite (3) as

\[
H_n = W(\tau_n) \alpha_n + \eta_n
\]

Relative to \( \alpha_{n,l} \), the tap delay \( \tau_{n,l} \) varies quite slowly \([7]\).

This means that for the multi-path channel, there exist two opposite characteristics – the slow variation of delays and the fast variation of amplitudes.

Through the fast Fourier transform (FFT) transform, the received signal at OFDM symbol \( n \), subcarrier \( k \) is represented as

\[
Y_n(k) = X_n(k)H_n(k) + W_n(k)
\]

where \( W_n(k) \) is the additive white Gaussian noise (AWGN) with variance \( \sigma_n^2 \), \( k = 0, 1, \ldots, N - 1 \). Note that the inter-carrier interference (ICI) is not considered here.

To estimate \( H_n \) defined in (4), we insert \( N_p \) pilot symbols \( X_{n,p}(k) \) among the \( N \) subcarriers under the comb pattern \([7]\).

Let \( X_{n,p}(k) \), \( H_{n,p}(k) \) and \( Y_{n,p}(k) \) denote the transmitted signal, channel response and received signal, respectively, at the pilot subcarriers, \( k = 0, 1, \ldots, N_p - 1 \).

## 3 Kalman-filter-based channel estimation in single-antenna scenarios

As shown in Fig. 1, the whole pilot-aided parametric channel estimation for \( H_n \) includes three steps: first, the channel responses at pilot subcarriers are calculated with the least-squares (LS) method; then, the parameters of each path are estimated and tracked to improve the accuracy of the estimation at pilot subcarriers; finally, the channel response at all the \( N \) subcarriers is obtained by interpolating or smoothing. The proposed method is applied in the second step, which is based on the consideration below.

(S1) Translating the estimation of channel parameters into an unconstrained minimisation problem for low-dimensional signal subspace.

(S2) Solving the unconstrained minimisation problem via subspace tracking by Kalman-filter.

In the following, we assume that the number of channel paths \( L \) is obtained via the minimum description length (MDL) method \([7]\) in advance.

### 3.1 Unconstrained minimisation problem for signal subspace

The LS estimation for \( H_{n,p}(k) \) is \( H_{LS,n,p}(k) = Y_{n,p}(k)/X_{n,p}(k) \) \([7]\). Omit subscript \( p \) and let \( H_{LS,n} = [H_{LS,n}(0) \quad H_{LS,n}(1) \quad \ldots \quad H_{LS,n}(N_p - 1)]^T \), then

\[
H_{LS,n} = W(\tau_n) \alpha_n + \eta_n
\]

Therein, \( N_p \times N \) Vandermonde matrix \( W(\tau_n) \) is the collection of the rows corresponding to pilot subcarriers in \( W(\tau_n) \). \( \eta_n \) is a vector term of AWGN with the variance of its entry as \( \sigma_n^2 \). As mentioned above, \( \tau \) varies much more slowly than \( \alpha_n \), so two other assumptions are given below.

**Assumption 3:** \( \tau_n \) is time-invariant for a sufficiently long time-scale, that is, \( \tau_n = \tau \) and \( W(\tau_n) = W(\tau) \).

**Assumption 4:** \( \tau_l \) are different. So \( W(\tau) \) has full column rank \( r = L \ll N_p \).

Introduce the singular value decomposition (SVD) as \( W(\tau) = U \Sigma V^H \), then, \( H_{LS,n} \) can be represented as

\[
H_{LS,n} = Ud_n + \eta_n
\]
where the $r \times 1$ vector $d_r \triangleq \Lambda V^H \alpha_r$. And the $N_p \times r$ matrix $U$ satisfies $U^H U = I_r$. We define the correlation matrix as

$$
\Phi = E \left( H_{LS,n} H_{LS,n}^H \right) \\
= U \Lambda V^H E(\alpha_r^2 \alpha_r^H) V \Lambda U^H + \sigma_n^2 I_{N_p},
$$

(8)

Therein, $\Sigma$ is a diagonal matrix. $U$ consists of $r$ eigenvectors corresponding to the main eigenvalues in $\Sigma$, and these eigenvectors span the signal subspace. So given the LS estimation result $H_{LS,n}$, the estimation of $W_r(x)$ is equivalent to tracking the signal subspace $U$. Here, the constant character of $U$ roots in the time-invariance of $\tau$, and just this constant character guarantees the validity of the optimisation problem in the following.

In order to obtain the minimum mean-square-error (MSE) of estimation, we define a primal cost function as

$$
J_0(U, d_n) = E \left\{ \| H_{LS,n} - Ud_n \|^2 \right\}
$$

(9)

Given $U$, minimising (9) yields the estimator of $d_n$ as

$$
\hat{d}_n = \arg \min_{d_n} E \left\{ \| H_{LS,n} - Ud_n \|^2 \right\} \\
= U^H H_{LS,n}
$$

(10)

Then, we obtain a new equivalent cost function as

$$
J(U) = E \left\{ \| H_{LS,n} - UU^H H_{LS,n} \|^2 \right\} \\
U \in \mathbb{C}^{N_p \times r}
$$

(11)

The estimator for $U$ is generated by

$$
\hat{U} = \arg \min_{U \in \mathbb{C}^{N_p \times r}} E \left\{ \| H_{LS,n} - UU^H H_{LS,n} \|^2 \right\}
$$

(12)

Based on (12), we can interpret the signal subspace tracking as a solution to an unconstrained minimisation problem. Then, the estimation of $\alpha_r$ and $\tau$ is converted into an adaptive process of solving an optimisation problem for $U$.

### 3.2 Subspace tracking with kalman filter

It has been proved that $\hat{U}$ is a stationary point of $J(U)$ (12), iff $\hat{U} = U_t Q$, where $U_t \in \mathbb{C}^{N_p \times N_p}$ contains any $r$ distinct eigenvectors of $\Phi$ and $Q \in \mathbb{C}^{r \times r}$ is an arbitrary unitary matrix. Moreover, $J(U)$ has a global minimum at which the column space of $U$ is equal to the subspace spanned by the $r$ dominant eigenvectors of $\Phi$, that is, the signal subspace of $\Phi$ (see Theorems 1, 2 in [13]).

Therefore a global convergence of solving $J(U)$ is guaranteed by tracking the signal subspace of $\Phi$. There exists a unique subspace spanned by $\hat{U}_{opt} \in \mathbb{C}^{N_p \times r}$ such that $U^H_{opt} U_{opt} = I_r$ and $U_{opt} = \arg \min_{U \in C^{N_p \times r}} J(U)$.

Expand the expression of $J(\hat{U}_{opt})$ as

$$
J(\hat{U}_{opt}) = E \left\{ H_{LS,n}^H H_{LS,n} \right\} - 2E \left\{ H_{LS,n}^H \hat{U}_{opt} \hat{U}_{opt}^H H_{LS,n} \right\} \\
+ E \left\{ H_{LS,n}^H \hat{U}_{opt} \hat{U}_{opt}^H \hat{U}_{opt} \hat{U}_{opt}^H H_{LS,n} \right\}
$$

(13)

Because of $H_{LS,n} = \hat{U}_{opt} d_n + \eta_n$, $\hat{U}_{opt} \hat{U}_{opt}^H = I_r$, $d_n$ and $\eta_n$ are uncorrelated, (13) can be rewritten as

$$
J(\hat{U}_{opt}) = E \left\{ \eta_n^H \eta_n \right\} - E \left\{ \eta_n^H \hat{U}_{opt} \hat{U}_{opt}^H \eta_n \right\} < N_p \sigma_n^2
$$

(14)

Since the existence of the optimal $\hat{U}_{opt}$, we can utilise Kalman-filter to approach $\hat{U}_{opt}$ adaptively. Denote

$$
\chi(n) \triangleq H_{LS,n}
$$

(15)

$$
\chi(n) \triangleq \hat{U}_{opt}(n) \chi(n)
$$

(16)

Next, upon the signal subspace we construct the state equation

$$
\hat{U}_{opt}(n + 1) = \hat{U}_{opt}(n)
$$

(17)

and the measurement equation

$$
\chi(n) = \hat{U}_{opt}(n) \chi(n) + \epsilon_{opt}(n)
$$

(18)

where $\epsilon_{opt}(n)$ is the optimal error vector when adopting $U_{opt}(n)$, and it satisfies

$$
E \left\{ \epsilon_{opt}(n) \epsilon_{opt}(n)^H \right\} = E \left\{ \| \chi(n) - \hat{U}_{opt}(n) \hat{U}_{opt}^H(n) \chi(n) \|^2 \right\}
$$

$$
= J(\hat{U}_{opt})
$$

(19)

In this way, derived from the state equation (17) and the measurement equation (18), the channel estimation algorithm based on the subspace tracking by Kalman-filter is proposed, as presented in Table 1. Note that the ultimate $\hat{U}_{opt}$ in this algorithm is not fixed, that is, there exists an ambiguity of unitary matrix $Q$ such that $\hat{U}_{opt} = U_t Q$, where $U_t$ is the base of signal subspace of $\Phi$.

In order to accelerate the convergence, we update $\xi(n)$ for each OFDM symbol, $c_1 = 10$ and $c_2 = 0.75$ are adjustable constant factors obtained in trial.

Essentially, this Kalman-filter based channel estimation method aims to obtain the signal subspace by which the minimisation of (12) can be achieved. Because it only focuses on the low-dimensional signal subspace and applies tracking techniques to avoid matrix inversions or decompositions, it enjoys a quite low complexity. The numbers of multiplication operations within it are shown in Table 1. Obviously, the complexity of the tracking

| Table 1: Proposed Kalman-filter based channel estimation algorithm in single-antenna scenarios |
|----------------------------------|----------------------------------|----------------------------------|
| Initialisation                  |                                  |                                  |
| $K(1, 0) = l_t, \ U_{opt}(0) = l_0, \xi(0) = c_1$. |                                  |                                  |
| For each OFDM symbol $n = 1, 2, \ldots$. | Num. of multiplication operations |                                  |
| $x(n) = H_{LS,n}$               |                                  |                                  |
| $y(n) = U_{opt}(n - 1) x(n);$    | $N_p r$                         |                                  |
| $g(n) = K(n, n - 1) y(n)^r y(n)^r$| $2r^2 + r$                      |                                  |
| $K(n, n - 1) = K(n, n - 1) - g(n)$ | $r^2 + r$                       |                                  |
| $\hat{y}(n) = K(n, n - 1);$     |                                  |                                  |
| $U_{opt}(n) = U_{opt}(n - 1) + x(n);$ | $2N_p r$                      |                                  |
| $- U_{opt}(n - 1) y(n)^r g_o(n);$|                                  |                                  |
| Estimator output:               |                                  |                                  |
| $H_{kalman}(n) = U_{opt}(n) U_{opt}(n) x(n);$ | $2N_p r$                      |                                  |
| $\xi(n) = c_2 \cdot \text{norm} H_{kalman}(n)$ | $N_p$                         |                                  |
| $\xi(n) = x(n)$                 |                                  |                                  |

End
operations for each OFDM symbol is as low as \(O(N_pr)\),
comparing with \(O(N_p^2)\) of the QR-RLS method [7, 8].
With the estimator output \(\hat{H}_{\text{Kalman}}(t)\),
we finally apply an interpolation among the frequency-time grid.

4 Extension to multi-antenna scenarios

Here, we consider the extension of the Kalman-filter-based
method to the multi-antenna scenarios. In this case, the
channel at each receive antenna is the superposition of
those from all the transmit antennas. Thus, considering an
arbitrary receive antenna and \(N_t\) transmit antennas totally,
(4) should be rewritten as

\[
\hat{H}_n = \sum_{m=1}^{N_t} W(\tau^{(m)})\alpha_n^{(m)} + \eta_n
\]

(20)

where \(\tau^{(m)}\) and \(\alpha_n^{(m)}\) are the delay vector and the amplitude
vector of transmit antenna \(m\), respectively.

Traditionally, in the pilot-aided channel estimation for
multi-antenna OFDM systems, different transmit antennas
are allocated with exclusive pilot-symbol subcarrier
resources, so as to guarantee the interference-free among
different transmit antennas when carrying out channel esti-
mations. However, a large number of subcarriers are wasted
in this way. In fact, a delicate pilot-symbol design could
help to reduce the overhead of multi-antenna pilot
signalling. Here, we consider the extension of the Kalman-filter-based
interpolation among the frequency-time grid.

Thus, we can construct the interference-free LS estimation
results for transmit antennas \(m\) and \(m'\) as

\[
H_{LS,n} = \sum_{m=1}^{N_t} U^{(m)} d_n^{(m)} + \eta_n
\]

(22)

where \(d_n^{(m)} \triangleq \Lambda^{(m)} v^{(m)H} \alpha_n^{(m)}\).

Imagine that if the columns spaces of all \(\{U^{(m)}\}_{m=1}^{N_t}\)
are mutually orthogonal, these \(U^{(m)} d_n^{(m)}\) for different \(m\) can
be picked out directly from \(H_{LS,n}\). In this way, allocating
these transmit antennas with the same pilot-symbol subcar-
riers is feasible and high efficient.

Consider any two columns of the matrices \(W_p(\tau^{(m)})\)\(m=1,2\), denoted by \(w_{p,1}\) and \(w_{p,2}\), which correspond to \(\tau_1\) and \(\tau_2\). The absolute of their inner-product can be expressed as

\[
|w_{p,1}^H w_{p,2}| = \left| \frac{1}{N_p} \sum_{k=0}^{N_p-1} e^{j2\pi kr_1/N_p} e^{-j2\pi kr_2/N_p} \right|
= \frac{1}{N_p} \left| \sin \pi \Delta \tau / N_p \right|
\]

(23)

where \(\Delta \tau \triangleq \tau_1 - \tau_2\). Obviously, if \(\Delta \tau / T\) is an integer,
\(|w_{p,1}^H w_{p,2}|\) equals zero; otherwise, if \(\Delta \tau / T\) tends to \(\pm N_p/2,\)
\(|w_{p,1}^H w_{p,2}|\) tends to \(1/N_p\). In these two cases, \(w_{p,1}\) and \(w_{p,2}\)
can be thought to be absolutely orthogonal or approximately
orthogonal, respectively.

In the following, we take the example of two transmit
antennas \(m\) and \(m'\). If all the pairs of any column of \(W(\tau^{(m)}))\sqrt{N_p}\) and any column of \(W(\tau^{(m')})\sqrt{N_p}\) satisfy either of the two conditions above, \(W(\tau^{(m)}))\sqrt{N_p}\) and \(W(\tau^{(m')})\sqrt{N_p}\) can be thought to be orthogonal, and so are
their signal subspaces \(U^{(m)}\) and \(U^{(m')}\). In our approach, a circular factor \(\hat{\tau}\) such that
\(\hat{\tau} > \text{max}(\tau^{(m)}/N_p)\) is given. Assume the pilot symbols of transmit
antenna \(m\) are \(X^{(m)}(k), k = 0 \sim N_p-1\), then, the pilot symbols of transmit antenna \(m'\) are circled as

\[
x^{(m')}_n = x^{(m)}_n e^{-j2\pi nk/N_p} k = 0 \sim N_p - 1
\]

(24)

In this way, at the receive antenna, by
\(H_{LS,n} = Y_n(\hat{\tau})/X^{(m)}(k)\), it holds

\[
H_{LS,n} = \sum_{m=1}^{N_t} U^{(m)} d_n^{(m)} + \eta_n
\]

(25)

where \(\tau^{(m')}\) is the equivalent path delays of transmit antenna \(m'\),
which is comprised the entries

\[
\tau^{(m')} = \tau^{(m)} + \hat{\tau} \quad \ell = 0 \sim L - 1
\]

(26)

The effect of the circular factor is illustrated in Fig. 2.
Obviously, the equivalent channel paths at the receive
antenna is the superposition of the original channel paths
form transmit antenna \(m\) and the shifted ones from transmit
antenna \(m'\). Note that the normalised value of path delays
has the module of \(N_p\), that is, if two normalised values are
equal modulo-\(N_p\), they would superpose each other.

Normally, the number of pilot symbols is greatly larger
than the normalised value of the maximum delay, that is,

\[
P_r \gg c \text{ max } \tau^{(m)}/T, \quad \exists c \geq 2, c \in \mathbb{N} \quad \forall m
\]

(27)

In this case, let \(\hat{\tau} = N_p/Tc\), then the subspaces \(U^{(m)}\) and \(U^{(m')}\)
can be thought to be approximately orthogonal. Thus, we can construct the interference-free LS estimation
for transmit antennas \(m\) and \(m'\) as

\[
H_{LS,n} = H_{LS,n} - U^{(m)} U^{(m)H} H_{LS,n}/28
\]

(28)

\[
H_{LS,n} = H_{LS,n} - U^{(m')} U^{(m')H} H_{LS,n}/29
\]

(29)

For transmit antenna \(t = m, m'\), let \(X^{(t)}(n) \triangleq H_{LS,n}^{(t)}\) and
\(Y^{(t)}(n) \triangleq U^{(t)opt}(n)X^{(t)}(n)\), plunge them into the measurement equation (18), and then carry out the Kalman-filter based
subspace tracking method. In this way, \(U^{(t)opt}\) and \(H^{(t)opt}\) can
be tracked and estimated simultaneously. The general
Kalman-filter based channel estimation algorithm for multi-
antenna scenarios is presented in Table 2.
and quadrature phase-shift keying (QPSK) modulation.

Simulations and discussions

Channel estimation can be obtained at the pilot-symbol subcarriers. In this way, the same accuracy of symbol subcarriers and employing the same total transmit power can be guaranteed only when the comb pattern [7]. The system bandwidth is 10 MHz and the carrier-frequency is $f_c = 3$ GHz. ITU-R M.1225 Vehicular test environment Channel A is applied as channel model, in which six paths are assumed with fixed delays and variational amplitudes [14]. Here, we simulate two channel cases, that is, block fading and time variation because of Doppler frequency spread. In block fading channels, there exists no time variation within an OFDM symbol duration, but the amplitudes of each path between two neighbouring symbols vary independently. Hence, there is no ICI in this case. In another channel case, which is more practical, the time variation because of Doppler frequency spread not only occurs between adjacent OFDM symbols, but also within one OFDM symbol. Therefore ICI exists at each subcarrier. By comparison, we show the QR-RLS method [7, 8], which performs well at the cost of high complexity. $f_d$ denotes the maximum Doppler frequency spread.

5.1 Block fading channels

First, Figs. 3 and 4 show the case of block fading channels. In this case, only the noise would influence the channel estimation at pilot subcarriers. In Fig. 3, the MSE of channel estimation against OFDM symbol index is considered. These convergence curves show that with Kalman-filter, the MSE fast approaches a quite low value, called by the post-convergence value, as well as QR-RLS.

Fig. 4 shows the curves of post-convergence MSE against signal-to-noise ratio (SNR). Because no ICI exists, the logarithmic values of MSE descend as SNR increases linearly. The curves of Kalman-filter and QR-RLS superpose each other, with the same convergence speed and error floor, and both of them outperform LS estimation nearly one order of magnitude. Essentially, this performance gain

Table 2: Proposed Kalman-filter based channel estimation algorithm in multi-antenna scenarios

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>LS</th>
<th>Kalman-filter</th>
<th>QR-RLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1.2</td>
<td>0.8</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Remarks:

- The dominant merit of this approach is reducing the amount of the total necessary pilot-symbol subcarriers by allocating a group of transmit antennas with the same pilot-symbol subcarriers, but maintaining the same estimation performance as by allocating them with different pilot-symbol subcarriers. The order of pilot-symbol sharing depends on $c$. However, the cost of this approach is limiting the maximum delay of multi-path channels.
- The premise of interference-free between $H^m_{LS, n}$ and $H^n_{LS, n}$ is the orthogonality between $U^m$ and $U^n$. However, this orthogonality could be guaranteed only when $\hat{U}^m_{opt}$ and $\hat{U}^n_{opt}$ are basically tracked. Therefore $\hat{U}^m_{opt}(n)$ and $\hat{U}^n_{opt}(n)$ should be initialised by allocating different pilot-symbol subcarriers to transmit antennas $m$ and $n$ in the first stage. Henceforth, the channel parameters could be tracked efficiently by the approaches (28) and (29).
- Another idea of multiple-antenna channel estimation is allocating these transmit antennas with different $N_p/c$ pilot-symbol subcarriers and employing the same total transmit power as by allocating them with the shared $N_p$ pilot-symbol subcarriers. In this way, the same accuracy of channel estimation can be obtained at the pilot-symbol subcarriers, but the pilot-symbol sharing approach can benefit greatly in the following step of interpolation.

5 Simulations and discussions

We consider an OFDM system with $N = 1024$ subcarriers and quadrature phase-shift keying (QPSK) modulation without coding. $N_p = 128$ pilot subcarriers are inserted in the comb pattern [7]. The system bandwidth is $B = 10$ MHz and the carrier-frequency is $f_c = 3$ GHz.
comes from the tracking of the low-dimensional signal subspace. The ratio of the decreasing MSE is equal to $L/N_p$, that is, the ratio of the number of paths to the number of pilot subcarriers. Thus, the more pilot subcarriers, the better channel estimation performances.

5.2 Channels with Doppler frequency spread

Then, Figs. 5–7 show the channel case with Doppler frequency spread. Here, $h_n$ in (6) consists of AWGN terms and ICI terms. With $f_d$ increases, the ICI tends to be serious, so that the channel estimation performance tends to deteriorate. In addition, the value of $f_d$ also determines the rate of channel time variance. From (2), whatever $f_d$ is, the correlation between neighbouring OFDM blocks exists. The simulation is performed in two kinds of channel environment shown in Table 3. The coherent time is defined as $T_d = 0.423/f_d$ [15]. Obviously, with the increase of $f_d$, $T_d$ decreases, and so does $T_d/T_s$.

In Fig. 5, the MSE of channel estimation against the OFDM symbol index with 200 Hz Doppler frequency spread is considered. With the Kalman-filter based method, the ultimate capture of $\hat{U}_{opt}$ can be guaranteed, although there may exist some fluctuant phenomenon in the beginning. Since in Kalman-filter, the update of $U_{opt}(n)$ is via low-complexity linear addition, it requires a certain decorrelation between adjacent samples of $H_{LS,n}$. Contrarily, with the QR-RLS method, the unitary character of the filter can be guaranteed by high-complexity QR decompositions in each update step, and hence, its convergence is not so sensitive to the channel samples' decorrelation. As shown in Table 3, at $f_d = 200$ Hz, $T_d/T_s \approx 21$, which means the amplitudes of channel paths vary so slowly that the correlation within about 21 neighbouring OFDM symbols still keeps high. Therefore the correlation between adjacent symbols is too high for the samples of $H_{LS,n}$ to provide enough information to accelerate the convergence of $U_{opt}(n)$ to $U_{opt}$ in the Kalman-filter based method. So the MSE curve fluctuates at the approximate period of 21 symbols several times before the capture of $U_{opt}$. Apparently, the fluctuant period approximately fit to the value of $T_d/T_s$ in Table 3.

With the increase of $f_d$, the correlation in neighbouring symbols grows lower. Thus, although severe ICI causes the worse performance of LS estimation and the higher post-convergence MSE floor, the convergence speed becomes faster. When $f_d = 500$ Hz as shown in Fig. 6, for the Kalman-filter curve, there is no fluctuant phenomenon, and it shows nearly the same convergence as the QR-RLS curve. Thereby, the Kalman-filter based method is especially suitable for the channel environment with large Doppler frequency spread.

Fig. 7 shows the curves of post-convergence MSE against SNR at different $f_d$. It can be found that the Kalman-filter based method and the QR-RLS method, with their curves superposing each other, both reduce the MSE of channel estimation results nearly one order of magnitude less than

<table>
<thead>
<tr>
<th>Vehicular velocity, km/h</th>
<th>$f_d$, Hz</th>
<th>$T_d$, ms</th>
<th>$T_d/T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>200</td>
<td>2.1</td>
<td>21</td>
</tr>
<tr>
<td>180</td>
<td>500</td>
<td>0.85</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Table 3: Setting of simulation parameters for Doppler frequency spread
Kalman-filter. The key of this method lies in the comprehension that the estimation of channel parameters can be translated into an unconstrained minimisation problem. Then the Kalman-filter based subspace tracking is applied to solve this optimisation problem, which consumes low computation complexity. Simulation results demonstrate that the proposed method performs well in both block fading channels and Doppler frequency spread channels. Further, the extension of the Kalman-filter based method to multi-antenna scenarios has been considered, where the channel estimation for multiple transmit antennas can be carried out with greatly reduced pilot-symbol overhead.

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8 References