Subspace Tracking based Channel Estimation with Rank Adaptation for OFDM Systems in Sparse Fading Channels

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Abstract—This paper presents a subspace tracking based channel estimation utilizing bi-iterative least-square (Bi-LS) rank adaptation method in sparse channel for OFDM systems. After modeling parameterized fading channel with sparse multipath, a general description of parametric channel estimation process based on subspace tracking is given. Then a simple bi-iterative least-square (Bi-LS) method is discussed with its capability of rank adaptation and low computational complexity. Finally, numerical simulation results show that Bi-LS method has better performance not only in subspace tracking, but also in subspace rank updating compared with other channel estimation techniques, especially when the multipath number is time variant or the sampling timing synchronization jitters.

I. INTRODUCTION

Recently, orthogonal frequency-division multiplexing (OFDM) has been one of the key techniques in wireless communication due to its superior anti-multipath performance. Within OFDM systems the accuracy of channel estimation will influence the performance of pre-coding at the transmitter, data detection at the receiver, and the Inter-Carrier Interference (ICI) mitigation technique; especially when the channel environment is time variant or the terminal has a high moving speed. Thus, the effective channel estimation method, aimed at estimating and tracking the changing OFDM channel, is worthy of investigation.

The traditional channel estimation methods based on zero-forcing (ZF) and minimum mean-squared-error (MMSE) criteria are not satisfactory. Additionally, a class of methods based on subspace tracking like least mean-square (LMS) and Kalman filtering algorithm [1] seem to lack the capability of rank adaptation. Thus, the goal of this paper is to find a subspace tracking-based channel estimation method in sparse fading channels. The algorithm focused on should have good rank updating performance combined with low complexity. A sparse fading channel means the effective path number of the multipath is low, (usually 2–6), and the time delay of the channel changes slowly, in accordance with the macro cellular system, where the antennas of the base stations are very high.

Channel estimation methods based on pilot [2][3] have been universally studied; the concept that comb-like pilots uniformly distribute through the frequency domain is the optimal pilot allocation in the mean square error (MSE) sense [3]. Accordingly, the channel estimation method based on subspace tracking using comb pilots will be studied.

This paper is organized as follows; Section II introduces the parameterized sparse fading channel model in OFDM systems and a general framework of subspace tracking based channel estimation method is described. In Section III the specific Bi-LS method is presented and its computational complexity is analyzed. In Section VI the performance of Bi-LS subspace tracking algorithm for tracking the sparse fading channel with delay jitter and multipath number variant (the rank of the subspace is changing) is demonstrated comparatively. Section V concludes the paper.

II. SYSTEM MODEL AND SUBSPACE TRACKING BASED CHANNEL ESTIMATION

A. System Model

Consider OFDM systems with \( N \) subcarriers. A time-variant multipath channel can be modeled as a combination of weighted impulse responses with differentiated delay distribution:

\[
h(t, \tau) = \sum_{i=0}^{L-1} \alpha_i(t) \delta(\tau - \tau_i)
\]

where the \( \tau \)th path can be parameterized as a complex gain \( \alpha_i(t) \) and a time delay \( \tau_i \). These two parameters should satisfy the following assumptions:

A1) The maximum multipath delay is no longer than the cyclic prefix, i.e. there is no ISI in OFDM systems.

A2) Any complex gain \( \alpha_i(t) \) is quasi-static during one OFDM symbol transmission, but can vary from symbol to symbol, i.e. block fading.

A3) The total number of multipath \( L \) and any \( \tau_i \) are constant during the OFDM symbol time considered.

Assumption (A2) and (A3) corroborate two features of the parametric channel model: the amplitude of the channel

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The response is varying fast while the multipath delay is varying relatively slowly. This study of the subspace tracking based channel estimation method is based on these two assumptions.

The matrix description of the multipath OFDM channels can be derived as follows:

Define \( \alpha_{n,j} \) and \( \tau_{n,j} \).

The channel response on the \( k \)th subcarrier of the \( n \)th OFDM symbol of duration \( T_s = NT + T_{CP} \), \( T = 1/B \) where \( B \) is the system bandwidth and \( N \) the total number of carriers, \( T_{CP} \) cyclic prefix duration) can be written as:

\[
H_n(k) = \sum_{l=0}^{L-1} \alpha_{n,l} e^{-j2\pi l k / N}, k = 0,1,...,N-1. \tag{1}
\]

Define

\[
\alpha_n = [\alpha_{n,0} \alpha_{n,1} ... \alpha_{n,L-1}]^T,
\]

\[
\tau_n = [\tau_{n,0} \tau_{n,1} ... \tau_{n,L-1}]^T,
\]

\[
H_{n,p} = [H_n(0) H_n(1) ... H_n(N-1)]^T,
\]

\[
W(\tau_n) = [e^{-j2\pi l k / N}]_{l=0,...,N-1,j=0,...,L-1}.
\]

And \( W(\tau_n) \) is Vander monde matrix with dimension \( N \times L \). Therefore, (1) may be written as follow:

\[
H_n = W(\tau_n)\alpha_n \tag{1}
\]

According to assumption (A3), \( W(\tau_n) \) is the full column rank \( L \), which is less than the length of the pilot. After FFT transformation at the receiver, the received signal on the \( k \)th subcarrier can be represented as below:

\[
Y_n(k) = X_n(k)H_n(k) + N_n(k) \tag{2}
\]

where \( N_n(k) \) is the additive white Gaussian noise with the variance \( \sigma_n^2, k = 0,1,...,N-1 \).

In order to estimate \( H_n \), we inserted \( N_p \) pilots \( X_n,p(k) \) uniformly out of the \( N \)-subcarriers with comb pattern. \( X_n,p(k) \), \( H_n,p(k) \) and \( Y_n,p(k) \) represent the sending signal, corresponding channel response and the received signal respectively: \( k = 0,D,...,(N_p-1)D, D = N / N_p \) is the pilot separation.

Next, the channel estimation algorithm based on multipath delay subspace tracking under the model suggested by (1) and (2) is derived.

**B. Subspace tracking based channel estimation for OFDM systems**

![Figure 1](image-url)

Figure 1 The framework of subspace tracking based parametric channel estimation method

Figure 1 depicts the three main procedures in channel estimation based on multipath delay subspace tracking: First, the channel response at the pilot location is estimated by the Least Square (LS) method. Second, a group of multipath parameters, like the number of the multipath and time delay subspace parameters are estimated and tracked respectively for improved precision. In addition and contrary to the method proposed in [1], the rank of the subspace is estimated adaptively. Finally, the whole channel response over the \( N \)-subcarrier can be obtained through interpolation and smoothness. Here, the second procedure is the main focus, which consists of the following steps:

**Step 1** Adopt a rank estimation algorithm and derive the dimension of the matrix \( W(\tau_n) \), which is corresponding to the delay subspace.

**Step 2** Convert the problem of parametric channel estimation into an unrestrained optimization over a low-rank time-delay subspace.

**Step 3** Utilize an adaptive algorithm to solve this unconstrained optimization problem while tracking the changes within subspaces.

**Step 4** Project the channel estimation result derived by LS onto the delay subspace obtained by tracking to obtain an enhanced channel estimation at the pilot location.

Assume the number of the multipath \( L \) and the rank of the subspace \( k \) is obtained by the minimum description length (MDL) algorithm [3] and formulate the three procedures outlined above. The adaptive algorithm of subspace tracking and rank updating will be further studied in the next section.

First, adopt these LS criteria to get the channel response \( H_{n,p}(k) \) at the pilot location.

\[
H_{LS,n} = [H_{LS,n,p}(0) H_{LS,n,p}(0) ... H_{LS,n,p}(N_p-1)D]^T
\]

\[
\bar{X}_{n,p} = [X_{n,p}(0) X_{n,p}(D) ... X_{n,p}(N_p-1)D]^T
\]

\[
\bar{Y}_{n,p} = [Y_{n,p}(0) Y_{n,p}(D) ... Y_{n,p}(N_p-1)D]^T
\]

\[
\bar{N}_{n,p} = [N_{n,p}(0) N_{n,p}(D) ... N_{n,p}(N_p-1)D]^T
\]

where \( H_{LS,n} \), \( \bar{X}_{n,p} \), \( \bar{Y}_{n,p} \), \( \bar{N}_{n,p} \) represent the channel response, sending pilot, received pilot and additive Gaussian Noise at the pilot location in frequency domain, respectively. Thus, \( \bar{H}_{LS,n} = \text{diag}(\bar{X}_{n,p})^{-1} \cdot \bar{Y}_{n,p} \)

\[
\bar{H}_{LS,n} = \bar{H}_n + \text{diag}(\bar{X}_{n,p})^{-1} \cdot \bar{N}_{n,p} \tag{3}
\]

\[
W_p(\tau_n) = \bar{U} \bar{A} \bar{V}^H
\]

Thus \( \bar{H}_{LS,n} \) can be written as:
$\mathbf{H}_{LS,n} = \mathbf{U}d_n + \mathbf{E}_{LS,n}$

where the \( r \) dimension column vector \( d_n \) is defined as:

\[ d_n = \mathbf{A}^H \mathbf{a}_n \]

The \( N_p \times r \) matrix \( \mathbf{U} \) satisfy \( \mathbf{U}^H \mathbf{U} = \mathbf{I}_r \).

The correlation matrix is thusly defined below:

\[ \phi = E(\mathbf{H}_{LS,n} \mathbf{H}_{LS,n}^H) = \mathbf{U} \mathbf{A} \mathbf{A}^H E(\mathbf{a}_n \mathbf{a}_n^H) \mathbf{A} \mathbf{A}^H \mathbf{U}^H + \sigma_n^2 \mathbf{I}_{N_p} \]

\[ = \mathbf{U} \mathbf{U}^H + \sigma_n^2 \mathbf{I}_{N_p} \]

where \( \Sigma \) is a diagonal matrix, \( \mathbf{U} \) consists of \( r \)-dominant eigenvectors corresponding to the \( r \)-dominant eigenvalues of \( \Sigma \). These \( r \)-dominant eigenvectors span the signal subspace of the delay vector. Thus, given the LS estimation result \( \mathbf{H}_{LS,n} \), the estimation of \( W_{\mathbf{y}}(\mathbf{r}) \) is equivalent to the tracking of the signal subspace \( \mathbf{U} \).

In order to obtain the MSE of the channel estimation, the cost-function is defined below:

\[ J_0(\mathbf{U}, d_n) = E\{ || \mathbf{H}_{LS,n} - \mathbf{U}d_n ||^2 \} \]  
\[ (4) \]

Given \( \mathbf{U} \), (4) may be minimized, and estimation of \( d_n \) may be derived as follows:

\[ d_n = \arg \min_{d_n} E\{ || \mathbf{H}_{LS,n} - \mathbf{U}d_n ||^2 \} = \mathbf{U}^H \mathbf{H}_{LS,n} \]

Thus, the equivalent cost function of (4) can be found

\[ J(\mathbf{U}) = E\{ || \mathbf{H}_{LS,n} - \mathbf{U} \mathbf{U}^H \mathbf{H}_{LS,n} d_n ||^2 \} \], \( \mathbf{U} \in \mathbb{C}^{N_r \times N_p} \)

The estimation of \( \mathbf{U} \) is:

\[ \mathbf{U} = \arg \min_{\mathbf{U} \in \mathbb{C}^{N_r \times N_p}} E\{ || \mathbf{H}_{LS,n} - \mathbf{U} \mathbf{U}^H \mathbf{H}_{LS,n} d_n ||^2 \} \],  
\[ (5) \]

Accordingly, the estimated channel response at the pilot location is:

\[ \mathbf{H}_{ST,n} = \mathbf{U} \mathbf{H}_{LS,n} \]  
\[ (6) \]

where, (6) can be viewed as projecting \( \mathbf{H}_{LS,n} \) on to the subspace \( \mathbf{U} \). Now we can take the tracking result \( \mathbf{U} \) as the solution to the unconstrained optimization problem and convert the estimation of \( \mathbf{a}_n \) and \( \tau_n \) into solving \( \mathbf{U} \) in the optimization problem adaptively.

Note that solving the optimization problem adaptively is actually solving the multipath delay subspace implicitly, which differs from the explicit method previously mentioned [4]. The disadvantage of the explicit method is that rank of the subspace should be known in advance. Therefore, it cannot be applied in a scenario in which the number of the multipath is changing. However, the implicit method suggested here can overcome this challenge.


III. BI-ITERATIVE LEAST SQUARE METHOD

A. Description of Bi-LS method

There are numerous subspace tracking-based channel estimation algorithms: Kalman filtering, projection approximation and subspace tracking (PAST)[6] and Least Mean Square (LMS) methods. They all have low complexity \( O(N_pr) \). However, they have a common drawback: they cannot estimate the rank of the subspace adaptively. The QR-RLS method adopted in [7] is capable of rank estimation. However, the complexity is \( O(N_p r^2) \) due to the QR decomposition it applies.

Therefore, it is necessary to explore an adaptive algorithm of multipath delay subspace tracking capable of rank estimation with low complexity, especially when the sum of the multipath is changing.

The Bi-LS subspace tracking algorithms suggested by Shan [8] possess both merits mentioned above. It is designed to construct the optimal low rank approximation of a matrix with linear complexity. Because the optimal low-rank matrix approximation carries all the information of the subspace of the underlying vector sequence, the Bi-LS method is naturally useful for subspace tracking. However, Bi-LS method is derived from Bi-SVD algorithm in which the SVD is implemented by QR decomposition; in order to further reduce the complexity, Bi-LS algorithms use the Givens transformation to realize triangularization of the matrix.

Here this method is expanded into the channel estimation of sparse fading channels in OFDM systems, detailed formulation can be found in [8].

The main process is summarized below:

Initialization:

Set \( r_{max} > r \), \( U_{\text{temp}}(0) = [I_{r_{max}} 0]^T \), \( R(0) = I_{r_{max}} \), \( \rho(0) = 0 \), \( \alpha, \beta \).

For \( n = 1, 2, \ldots \),

Input: \( \mathbf{x}(n) = \mathbf{H}_{LS,n} \);

Bi-LS subspace tracking:

\[ y(n) = \mathbf{U}_{\text{temp}}(n-1) \mathbf{x}(n) ; \]

\[ R(n) = G(n) \begin{bmatrix} \alpha^n R(n-1) & 0 \\ 0 & (1-\alpha)^{n/2} y(n)^H(n) \end{bmatrix} ; \]

\[ q(n) = [0, \ldots, 0 1] G^n(n) ; \]

\[ x_1(n) = \mathbf{x}(n) - \mathbf{U}_{\text{temp}}(n-1) y(n) ; \]

Solve \( \tilde{q}(n) \), according to \( R(n) \tilde{q}(n) = q(n) ; \)

\[ \mathbf{U}_{\text{temp}}^H(n) = \mathbf{U}_{\text{temp}}^H(n-1) + (1-\alpha)^{n/2} x_1(n) \tilde{q}_H(n) ; \]

Adaptive rank estimation:

\[ \hat{\lambda}_i = [R(n)]_{ii}, i = 1, 2, \ldots, r_{\text{max}} ; \]

\[ p(n) = \alpha p(n) + \frac{1-\alpha}{N_p} \text{tr} \{ \mathbf{x}(n) y(n) \} ; \]

\[ \sigma^2 = \frac{N_p - r_{\text{max}}}{N_p - r_{\text{max}}} \rho(n) - \frac{1}{N_p - r_{\text{max}}} \text{tr} \{ R(n) \} ; \]

\[ r_n = \text{card} \{ \hat{\lambda}_i : \hat{\lambda}_i > \beta \sigma^2 \} ; \]
where, \( r_{\text{max}} \) is the maximum dimension of the subspace set in advance, \( \alpha \) is the forgetting factor and \( \beta \) is the threshold parameter of rank estimation. \( G(n) \) is the product of a series of Givens transformation matrices with the last row of the matrix \( \alpha^{1/2} R(n-1) \) a zero vector. \( tr\{\cdot\} \) represents the trace of the matrix while \( card\{\cdot\} \) represents the number of the elements in the set. \( \tilde{U}_{\text{temp}}(n) \) and \( R(n) \) are \( N_p \times r_{\text{max}} \) subspace matrix and \( r_{\text{max}} \times r_{\text{max}} \) upper triangular matrix, respectively. Both are updated continuously during the tracking process.

After \( \tilde{H}_{\text{Bi-LS}}(n) \) is derived using the method described above, the adoption of some interpolation method is needed to accomplish the channel estimation on all the subcarriers in the whole frequency domain.

### B. Convergence performance and complexity comparison

In this section, the error of the channel estimation at the pilot subcarrier using enhanced subspace tracking with the comparison of direct LS method will be discussed.

According to (3), the mean square error (MSE) is:

\[
\text{MSE}_{\text{LS}} = E[tr(\tilde{E}_{\text{LS},n}^H \tilde{E}_{\text{LS},n}^H)] / N_p = \frac{1}{SNR_p},
\]

where \( SNR_p \) is the average SNR at the pilot subcarriers. Here, the assumption is that the noise and the channel is independent, and the estimation error and channel is also independent i.e.;

\[
E[\tilde{N}_n^H \tilde{H}_{\text{a}}] = 0 \text{ and } E[\tilde{E}_{\text{LS},n}^H \tilde{H}_{\text{a}}] = 0.
\]

For the subspace tracking based methods, the asymptotic performance \( (n \to \infty) \) is considered, which means the number of the multipath is invariant and the subspace tracking is convergent \( (r=r) \). Thus,

\[
\tilde{H}_{\text{ST}}(n) \to UU^H \tilde{H}_{\text{LS},n} = U(d_a + U^H \tilde{E}_{\text{LS},n}),
\]

The corresponding error vector \( \tilde{E}_{\text{ST}}(n) \) is:

\[
\tilde{E}_{\text{ST}}(n) \to UU^H \tilde{E}_{\text{LS},n},
\]

Its mean square error is:

\[
\text{MSE}_{\text{ST}} = E[tr(\tilde{E}_{\text{LS},n}^H \tilde{E}_{\text{LS},n}^H)] / N_p \to \frac{r}{N_p \cdot SNR_p}
\]

Therefore, considering channel estimation error at the pilot subcarriers, the gain of the subspace tracking based method is \( N_p / r \).

#### IV. SIMULATION

In this section, the simulation results of Bi-LS subspace tracking based SISO channel estimation in OFDM systems is presented.

Consider an OFDM system with 1024 subcarriers, QPSK modulation, and without channel coding. The pilots are inserted every 8 subcarriers in frequency domain with 128 pilots in each OFDM symbol. Pilots and the signal are transmitted with the equal power; the system bandwidth is 10MHz, with a carrier frequency of 3GHz. The model of the channel is ITU-R M.1225 with 6 multipoles.

The channel response estimation at the pilot subcarriers using subspace tracking algorithms or LS is found first, then, using IFFT transformation, the channel response estimation over all the subcarriers is found.

#### A. Convergence performance

In this section the convergence performance of the Bi-LS algorithm, when the maximum Doppler shift is 200Hz is shown. The simulation result is displayed in Figure 2.

![Figure 2 MSE performance with Maximum Doppler Shift 200Hz](image-url)
From the figure 2, it can be seen that the Kalman algorithm has some fluctuation at the first tracking stage. One of the possible reasons is that the updating of $\hat{U}_{\text{opt}}(n)$ is implemented by simple linear operator rather than orthogonal transformation. In contrast, the QR transformation in QR-RLS and Givens transformation in Bi-LS can guarantee the orthogonality of the updating filter. Therefore, the convergence performance is not sensitive to the correlation between samples.

B. Rank tracking performance

In this part, the rank tracking performance when the number of the multipath is changing is considered. In the simulation, set $r_{\text{max}} = 8$, multipath number is 6 at first, after the 200 OFDM symbols duration, it becomes 5, and after another 200 OFDM symbols, it reverts to 6. Each set-path has equal power and SNR is 30dB. The simulation result is shown in Figure 3, where (a) depicts the MSE performance and (b) depicts rank tracking performance.

![Figure 3 Subspace tracking with the variant rank and Doppler 200Hz](image)

(a) The MSE of channel estimation (b) Rank estimation result

This figure depicts that when the multipath number decreases, the MSE of subspace tracking based channel estimation algorithm fluctuates modestly. In contrast, when the multipath number increases, the MSE performance aggravates due to the underestimation of the subspace rank, which is led by the lag-feature of the rank tracking. However, as the rank estimation increases in accuracy, the performance rises to normal, which verifies the efficiency of the suggested rank tracking algorithm.

C. Performance in time jitter

In this section, the performance of the suggested algorithm is tested when the delay jitters. Consider the stability of the system clock is 10ppm, which means when the sampling rate is 10MHz, there is about 0.1 sample jitters in each OFDM symbol. Therefore, a random jitter is added, which yields uniform distribution between [-0.01 0.01] to the multipath delay vector in every OFDM symbol. SNR is 30dB and the maximum Doppler shift is 200Hz. The result is illustrated in the Figure 4.

![Figure 4 Subspace tracking with delay jitter and Doppler 150Hz](image)

(a) the delay jitter (b) the delay varies continuously

It is clear that, in spite of the delay jitter, the subspace tracking based algorithm can track the multipath subspace effectively with subsequent increased performance in channel estimation.

V. CONCLUSION

In this paper, a new pilot-aid implicit multipath subspace tracking-based sparse-channel estimation method in OFDM systems has been proposed. It reformulates the channel estimation problem into an unconstrained optimization problem in a low rank signal subspace, acquiring both better performance and lower complexity. The Bi-LS subspace tracking method can track the subspace and estimate the rank effectively, even when the delay is jittering or the number of multipath is changing.

The extension of this algorithm to MIMO OFDM channels will be further studied.

REFERENCES


